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Estimation of Static Formation Temperatures in Geothermal and Oil Well by Inversion of Logged Temperatures

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ABSTRACT

Three inverse methods are used to obtain static formation temperatures in geothermal and oil wells. The methods include: (a) A least squares optimization algorithm or Levenberg-Marquardt method (MLM); (b) A Proportional Integral Control method (PI), and (c) An Artificial Intelligence method (IA). Results obtained using data from an oil well show that it is feasible to predict SFT reasonably by these methods however the IA method predicts lower temperatures. SFT’s obtained by the MLM and PI methods also compare well with simpler methods like the Horner and spherical-radial flow method.

1. Introduction

Knowledge of static formation temperatures (SFTs) is essential for many areas of engineering and scientific research, e.g., reservoir engineering; well completion and production logging; exploitation of hydrocarbons and geothermal energy; the study of groundwater and aquifer potential; evaluation of formation thermal conductivity; minerals, and study of the Earth’s evolution among others. SFTs in wellsbores can be inferred from temperature logging; empirical or analysis of fluid inclusions. SFTs can also be obtained by numerical simulation whereby logged temperatures during well drilling are reproduced (Kutasov et al., 1988; Ikeuchi et al., 1998, Saito et al., 1998, Wooley, 1980). Numerical simulation is a complex task and usually requires a great deal of information on drilling fluid composition, inlet fluid temperatures, fluid circulation rate and circulation losses, well geometry characteristics, geothermal gradient (a guess on the initial condition which is used to start the simulation), and thermophysical properties.

Drilling of a wellbore is essentially a transient process due to circulation (cooling) and shut-in (heating) processes. During well drilling, the formation temperature is perturbed from the original condition (SFT), which in practice is unknown. Thus, inverse heat transfer problems used to determine SFTs are based on directly measured quantities such as bottom-hole temperatures (BHT) or temperature logs. This is a typical inverse problem, in contrast to the direct problem, whereby the temperature field (BHT) is computed from a knowledge of the initial condition (SFT), that is, the inverse problem is associated with the reversal of the cause-effect sequence and consists of finding the unknown causes (SFT) of known consequences (BHT). The solution of an inverse problem is not straightforward and requires numerical techniques to stabilize the results of calculations. A commonly used algorithm for solving inverse problems is based on non-linear least squares and is known as the Levenberg-Marquardt algorithm (Marquardt 1963). More recently, Olea-Gonzalez (2007) included Artificial Intelligence (AI) and Proportional Integral control (PI) algorithms in inverse formulations to estimate SFTs.

The specific problem being investigated herein is the estimation of SFTs based on the inverse solution of a 2D fully-transient heat transfer problem with convective and conductive mechanisms using BHT measurements. An initial condition (SFT) is assumed and the temperature field, T(r,z,t) or BHT in the wellbore is obtained. These temperatures are compared with temperature logs (BHT) at different times and depths until both temperatures agree reasonably with each other. In the inversion of the logged temperatures to obtain SFTs, the assumed initial temperature or assumed initial condition and other fitting parameters are varied until the best match between logged and computed BHT are obtained. The last version of the initial condition is taken as the SFT. The inversion of logged temperatures is computed using three inverse algorithms: MLM, AI and PI.

2. Physical Model

General model considerations include cylindrical well geometry and axial and radial transient heat flow. Heat exchange with
the formation is convective if circulation losses exist or conductive after circulation stops. The rock is considered as an infinite medium while the formation zone invaded by drilling fluid is considered as a porous medium. Figure 1 shows the physical model under study.

3. Mathematical Model of the Well and Surrounding Formation

The estimation of temperatures in and around a well during circulation and shut-in conditions in the presence of lost circulation was performed by numerical simulation. For the simulation, the initial condition is an assumed temperature profile, which can be the geothermal gradient. The mathematical model considers transient convective heat transfer due to circulation losses to the rock surrounding the well, and consists of a set of partial differential equations describing the 2D transient heat transfer problem. Mass conservation considers incompressible flow in the axial \((r, z)\) and radial \((\varphi)\) directions. The solution considers the convective heat transfer effects that appear in the boundary conditions. The well formation interface is considered as a porous medium through which fluid may be lost or gained by the well (Figure 1). The fundamental assumptions of the model include: cylindrical geometry, isotropic rock formation, constant physical properties, negligible viscous dissipation and thermal expansion effects, and incompressible fluid. Model details are given by Garcia-Gutierrez et al. (2002) and García-Gutiérrez and Espinosa-Paredes (2005). With this, the governing equations and initial and boundary conditions are:

### Mathematical Formulation

The five heat flow regions are shown.

\[ \rho C_p \left( \frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + v_z \frac{\partial T}{\partial z} \right) = k \left( \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} \right) \]  

\[ \frac{1}{r} \frac{\partial (rv_r)}{\partial r} + \frac{\partial v_z}{\partial z} = 0 \]  

I.C. \[ T(r, z, t) = ? \] \[ \text{en} \quad t=0 \quad \forall \ r \ y \ z \]  

B.C.1 \[ -k \left( \frac{\partial T}{\partial r} \right) = h \left( T_s - T_m \right) \] \[ \text{en} \ A_i \ \forall t \]  

B.C.2 \[ T(r, z) = T_i \] \[ r_s \leq r < \infty, \ t > 0 \]  

B.C.3 \[ v_z = \left( \frac{W}{\rho A_i} \right) \] \[ \text{en} \ z = 0 \ \forall t \]  

B.C.4 \[ v_r = f (\phi, W, W_{loss}, P_m, A_i) \] \[ \text{en} A_i \ \forall t \]  

The initial condition given by equation (3) is unknown, and it is the aim of this work to test different methods to determine this initial condition. In this case, the particular solution is known and corresponds to the well logged temperatures. In the above equations \( T_i \) is solid temperature; \( T_m \) is drilling fluid temperature; \( A_i \) is interfacial area between the rock formation and the fluid; \( \phi \) is porosity; \( C_p \) is specific heat; \( k \) is thermal conductivity; \( h \) is heat transfer coefficient; and \( v_z \) and \( v_r \) are axial and radial flow velocities, respectively. In the formation, considered as a porous media, \( T \) represents the average temperature and \( k \) is the thermal conductivity of the medium, \( \phi \) is the formation porosity and \( A_i \) is the area from lateral flow, \( W_{loss} \) are the circulation losses. These equations defined in generic form the direct problem to be solved in order to obtain the temperature field in the well \( T(r, z, t) \), and the inverse formulation compares this field with the measured temperature field (BHT) to obtain the unknown initial condition or SFT. The differential equations described above were transformed into discrete equations using the finite differences technique in an implicit form. The resulting set of non-linear algebraic equations was then solved using an iterative method.

4. Inverse Methods

Three methods to estimate SFTs in inverse form are proposed herein. The first is a Levenberg-Marquardt numerical optimization technique based on least-squares (MLM method). The second is a feedback control structure based on the instantaneous model error feedback (the PI method) to reduce the error between logged and simulated temperatures for best SFT estimate. The third is an expert’s decision-making process with a cognitive model and fuzzy sets to model the agent’s reactive deliberative process (AI method).

4.1 Levenberg Marquardt, MLM, Method

The procedure for estimating SFTs by the MLM [Marquardt, 1963], method includes five parts: 1) Direct problem; 2) Inverse problem; 3) Iterative process; 4) Convergence criteria, and 5) the computational algorithm. The objective function in this method is defined as:
such that the sum of the squared residuals be a minimum, $Y_i$ and $T_i(P)$ are vectors that contain the logged and simulated well temperatures. Figure 2 shows the MLM flow diagram.

### 4.2 Proportional-Integral Control, PI, Method

The idea of the method is that the axial profile of the simulated logged-temperature tracks the axial profile of the measured logged temperatures (BHT) using the PI control approach. From the point of view of control theory the logged temperatures represent the set point. The PI control is used to estimate the SFT from logged ($T_{log}$) and simulated ($T_{sim}$) temperatures. The PI control in Laplace transform is given by:

$$PL = K_p e(s) \left[ 1 + \frac{1}{\tau_s s} \right]$$

where $K_p$ and $\tau_s$ correspond to the proportional gain and integral time, respectively, and represent the PI control adjusting parameters. The tracking instantaneous error is given by

$$e(s) = (T_{log} - T_{sim})$$

which is defined as the difference between the logged and simulated temperatures applying discrete PI control, then the iteration process with PI action can be represented by:

$$T_{SFT}(k+1) = T_{SFT}(k) + K_p \left[ e(k) + I(k+1) \right]$$

$$e(k) = T_{log}(k) - T_{sim}(k)$$

$$I(k+1) = \frac{1}{\tau_s} \int_0^k e(s) ds + \frac{\Delta t}{\tau_s} e(k)$$

Here $T_{SFT}(k+1)$ represents a new SFT value at $k+1$, and $\Delta t$ is the time step. These equations are applied for all the spatial grid points at $k+1$ iteration. Figure 2 illustrates the PI method.

### 4.3 Intelligence Artificial, AI, Method

Cognitive models are based on formulation of a human expert knowledge with uncertainty, which is expressed in terms of fuzzy rules. Mathematically speaking, an inverse heat transfer is solved in the environment, in order to achieve an emergent response of the system. The cognitive model that represents the expert’s decision allows consideration of the different situations that can occur in the environment, in order to achieve an emergent response of the system.

1. A set of existing temperatures (initially guessed SFTs) that will be modified as the iterative process approaches the solution. These temperatures are denoted in three different ways, according to the different stages of the process. Existing: proposal (in the beginning of the process), modified (during the iterations to converge) and existing (as a result of the process), as illustrated in Figure 3.
2. A set of logged temperatures obtained during well drilling and characterization.
3. A set of simulated temperatures, which are obtained using the previously discussed mathematical model, which represent the virtual environment.

The cognitive model that represents the expert’s decision allows consideration of the different situations that can occur in the environment.

Thus, it may be said that the temperature behavior in the well has been successfully modeled when the difference between simulated and logged temperatures is inside the human perception. In a well drilling example, the value of this temperature error is 5 degrees, which is normally accepted by the experts in this field. In other words, this difference would not affect a technical decision made by an expert. Since the expert works with ranges of values that depend on the absolute difference (denoted numerical variation) between two temperatures, $T_{Sim}$ and $T_{log}$, this difference can be classified as: very small, small, medium, big, and huge. The final objective is to determine an increment/decrement value to adjust $T_{Sim}$.

The last value of $T_{Sim}$ used in the last calculations becomes $T_{Sim}(k+1) = T_{Sim}(k) + \Delta T$, where $\Delta T$ is the difference between the simulated and logged temperatures.

Figure 2. Block diagram of feedback control in the PI method.
a part of the input data in the convergence process. More details on this method are given by Olea-Gonzalez (2007).

5. Results and Discussion

The estimation of SFT using the three inverse methods proposed herein was done using field data of oil well A. The well data is given in Table 1. The true reported SFTs for this well include a linear geothermal gradient of 3°C/100 m between 25°C at zero depth and 110°C at a depth of 3500 m.

Figure 4 shows SFTs obtained by the MLM, PI and IA methods. Also included are SFT estimations using simpler methods like the Horner method, the spherical-radial heat flow method of Ascencio et al. (1994); the Hassan et al. (1991), and the Kritikos and Kutasov (1988) method. As mentioned before the reported true SFT for this well is a linear gradient between 25°C at the wellhead and 110°C at 3500 m depth. From the figure it is observed that all methods, inverse and simpler analytical ones, follow a linear trend all the way from the wellhead to the bottom of the well. The exceptions to this behavior are described next. (1) The IA method exhibits cooler temperatures between 2200 m and total depth that is in the convective zone of this reservoir, while the other two inverse methods are not affected by convective effects. (2) At about 2200 m, the Kritikos and Kutasov (1988) method shows the highest temperature of all methods, and this fact appears to be related to the abrupt change in heat conductance of the well in changing from a cemented stage to the deeper uncovered part of the well. (3) At the well bottom, the temperatures predicted by the simpler analytical methods are in reversed order form that observed at 2200 m depth. At this depth, the SFTs predicted by the MLM and PI methods are somewhat higher than the SFTs predicted by the analytical methods while the SFT predicted by the IA method are the lowest of all.

6. Conclusions

Three methods, MLM, PIC and AI, were in an inverse heat transfer problem to estimate static formation temperatures from logged temperatures in a wellbore. The temperature behavior in the wellbore has been successfully modeled according to our results. The performance of the method is illustrated by means of the simulation of an oil well from the Gulf of Mexico. We found that the SFTs obtained with the three methods are closer to the true SFT that those obtained with simple analytical methods. SFT’s obtained by the MLM and PI methods also compare well with simpler methods like the Horner and spherical-radial flow method. Further applications and validations of the present methods include a wider data set from oil and geothermal wells.

7. References


