A Heuristic algorithm to solve the unit commitment problem for real-life large-scale power systems

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ABSTRACT

One of the main needs that power system operators around the world have is to solve complex Unit Commitment models for large-scale power systems in an acceptable computation time. This Paper presents an alternative Heuristic algorithm that successfully addresses this need. The Heuristic algorithm makes use of various optimization techniques such as Mixed Integer Linear Programming (MILP), Quadratic Programming (QP), Quadratically Constrained Programming (QCP), and Dynamic Programming (DP). CPLEX 12.2 is used as the main optimization engine for MILP, QP, and QCP. DP is an in-house algorithm used to obtain the commitment of Combined Cycle Plants (CCPs) when represented with the component-based model. This Heuristic algorithm combines the global optimality capabilities of MILP formulations with the highly detailed models available for CCPs using LR–DP formulations. The Heuristic algorithm introduced in this Paper is capable of solving up to 1-week scenarios with a 1-hour time window for the complex Mexican Power System.

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1. Introduction

The Unit Commitment (UC) problem is one of the most widely studied problems in Electrical Engineering. Over the last four decades, a number of different techniques have been proposed to solve it. Mixed Integer Programming (MIP) was first used to solve the unit commitment (UC) problem in [1]. The formulation is based on the definition of three sets of binary variables to model the start-up, shut-down, and on/off states of every generator for every time period. In [2], an alternative Mixed Integer Linear Programming (MILP) formulation is presented. Even though it only requires a single set of binary variables, one per unit per period, more constraints are formed. An alternative way to linearize the problem has been presented in [3]. Another linear formulation is presented in [4]. A basic idea for combining Lagrangian Relaxation (LR) and MIP techniques is presented in [5], where, in order to solve the problem faster, a lower bound for the UC problem is obtained by means of LR and then this lower bound is used when solving the UC problem with the IBM CPLEX optimizer [6]. Other techniques used are dynamic programming [7], pure LR methods [8–11], unit de-commitment, advanced priority listing [12], and Benders Decomposition [13].

Population-based techniques have also been used to solve the UC problem [14,15]. In evolutionary programming (EP) techniques, populations of contending solutions are evolved through random changes, competition, and selection to obtain a final solution. In [16], an algorithm is proposed in which an overall UC schedule is coded as a string of symbols and viewed as a candidate for reproduction. Initial populations of such candidates are randomly produced to form the basis of subsequent generations. In [17], an evolutionary algorithm (EA) with problem specific heuristics and genetic operators has been employed to solve the UC problem. The initial random population is seeded with good solutions using a priority list method in order to increase the speed of convergence and improve the efficiency of the algorithm. In [14], a two-level, two-objective optimization scheme based on EAs is proposed for solving the UC problem. At the low level, a coarsened UC problem is defined and solved using EAs to locate promising solutions at low cost. Promising solutions migrate upwards to be injected into the high level EA population for further refinement. In addition, at the high level, the scheduling horizon is partitioned in a small number of subperiods of time which are optimized iteratively using EAs. Genetic algorithms (GAs) are a general purpose optimization technique that is based on the principle of natural selection and natural genetics. Reference [18] introduces an application of a combined LR and GA method for the UC problem. The proposed
LR–GA incorporates GA into the LR method in order to update the Lagrangian multipliers and improve its performance. Ref. [19] introduces a new genetic operator, based on unit characteristic classification, along with intelligent techniques that generate initial populations.

Several heuristic methods have also been proposed in the literature. For instance, [20] presents and adaptation of the extended priority list method. This heuristic algorithm consists of two basic steps: in the first step, an initial UC solution is obtained by the priority list method disregarding the operational constraints. In the second step the UC obtained is modified using some heuristics in order to fulfill the operational constraints. Reference [21] proposes an algorithm that uses priority list based heuristics in the form of inference rules to find a suboptimal, and later on an optimal, schedule for a given load pattern. In [22] a Lagrangian relaxation approach for the solution of the short-term unit commitment problem in hydrothermal power-generation systems is presented. The proposed approach is based on a disaggregated Bundle method for the solution of the dual problem. The disaggregated Bundle method provides information that can be used for generating a feasible solution of the primal problem and for obtaining an optimal hydro scheduling. Reference [23] proposes an enhanced adaptive Lagrangian relaxation for the UC problem; it consists of adaptive LR and heuristic search. After the adaptive LR best feasible solution is obtained, the heuristic search consisting of unit substitution and unit de-commitment is used to fine tune the solution.

Power System Operators around the world have the need to solve detailed UC models for large scale Power Systems within a computation time that is acceptable for operation practices. Several operational, technical, and economical constraints must be considered in these detailed models. Although LR-based approaches are highly regarded in the academia as some of the most effective and suitable ways to solve large-scale UC problems, their ability to find primal feasible solutions decreases as the number and complexity of relaxed constraints increases. Recent reports show the ability of commercial optimization software to solve real-life UC problems based on MIP, MILP, and MIQCP formulations [24]. In fact, the tendency amongst power system operators around the world is to move from LR formulations to MI (L) P-based formulations, i.e., PJM, California ISO, ISO New England, MISO, NYISO (near future), and the Southwest Power Pool (SPP) in the USA; and Terna in Italy.

Of particular interest for power system operators is the accurate modeling of Combined Cycle Plants (CCPs). To this date, there are no detailed CCP models for MI (L) P that compare to the one introduced by the authors in [25]. The model introduced there is a component-based model that accurately represents start-up sequences, different stopping modes, and transitions between configurations and states. The approach in [25] is an LR–DP approach. The implementation of detailed component-based models for CCPs and Hybrid CCPs (HCCPs) with MI (L) P is no trivial matter and, for large-scale power systems, computation time and memory requirements are expected to be a problem.

Based on the above, the authors introduce a fast-solving Heuristic algorithm capable of handling UC problems for large-scale power systems. This Heuristic algorithm combines the global optimality capabilities of MI (L) P formulations with the highly detailed available models for (H) CCPs using LR–DP formulations. To achieve this, the Heuristic algorithm successfully combines MILP, QP, QCP and DP.

The remaining sections of this Paper are as follows: Section 2 briefly comments on the MIQCP model for the UC problem, and on the LR–DP approach for (H) CCPs used by the Heuristic algorithm. A detailed presentation of the Heuristic algorithm is made in Section 3. In Section 4, the solution approach taken by the Heuristic algorithm is compared to two other approaches based on MIQ (C) P in order to validate its solution quality. Afterwards, the Heuristic model is used to solve the Mexican Power System for 1-week planning horizons. Finally, Section 5 closes the Paper with relevant conclusions.

2. An MIQCP model for the UC Problem and an LR–DP component-based model for (H) CCPs

This section briefly comments on the MIQCP model used to represent the UC problem and on the LR–DP component-based model used to represent (H) CCPs. Both models have been previously introduced by the authors in [24,25], respectively. For the sake of continuity, the model introduced [24] is reproduced next without any further comments. The notation used in the model is described in Appendix (A).

### 2.1. MIQCP model for the UC problem

- **Objective Function:** Minimizes the variable generating costs, fixed start-up costs from either a cold stop or a hot stop of generating units, the cost of purchasing energy from Independent Power Producers, the cost of curtailing load from interruptible loads, the cost of shedding load, and the cost from violating the transmission limits in regional tie-lines and groups of tie-lines.

Two simplifications made in the MIQCP model are worth mentioning: The one is that (H) CCPs are represented using the aggregated model, and the other is that the start-up costs for both Thermal Conventional Units (TCUs) and (H) CCPs are considered fixed.

\[
\sum_{i \in I} \left\{ \sum_{u \in U} \left[ d_{ui} g_{ui}^2 + b_{ui} g_{ui} + c_{ui} r_{ui} + A F_{ui} r_{ui} + A C_{ui} r_{ui} \right] 
+ \sum_{i \in I} C P_{ui} g_{ui} + \sum_{i \in I} C I_{i} L_{ui} + C C X_{i} + \sum_{m \in M} P_{m} \left( f_{m,i} + f_{m,i}^* \right) \right\}.
\]

### Constraints:
- **Power balance**
  \[
  \sum_{u \in U} g_{ui} + \sum_{i \in I} L_{ui} + X_{i} = d_{i}, \quad \forall \ i \in I.
  \]
- **Spinning reserve**
  \[
  \sum_{u \in U} \left( g_{ui} - \beta_{ui} \right) 
+ \sum_{h \in H} \sum_{r \in R} \left( \beta_{ui} \sum_{h \in H} - \sum_{u \in U} \sum_{h \in H} \beta_{ui} g_{ui} \sigma_{ui} - g_{ui} \right) 
\geq \ L_{i}, \quad \forall \ i \in I.
  \]
- **Active power flow on regional tie-lines and groups of regional tie-lines**
  \[
  \sum_{u \in U} s e_{ui} + c e_{ui} - f_{xui} \leq f_{bmi}, \quad \forall \ m \in M, \quad i \in I.
  \]
  \[
  \sum_{u \in U} s e_{ui} + c e_{ui} + f_{xui} \geq -f_{bmi}, \quad \forall \ m \in M, \quad i \in I.
  \]
  \[
  \sum_{u \in U} s e_{ui} + c e_{ui} - f_{xui} \leq -f_{bmi}, \quad \forall \ n \in N, \quad i \in I.
  \]
  \[
  \sum_{u \in U} s e_{ui} + c e_{ui} + f_{xui} \geq -f_{bmi}, \quad \forall \ n \in N, \quad i \in I.
  \]
– Ramp-up and ramp-down capabilities of TCUs and (H) CCPs
  \[ g_{ul1} - g_{ul1} - \beta_{ul1} + \beta_{ul1} \leq \beta_{ul1}, \forall u \in U, i \in I, \]
  \[ b_{ul1} - g_{ul1} - \beta_{ul1} - \beta_{ul1} \leq \beta_{ul1}, \forall u \in U, i \in I. \]

– Quadratic fuel consumption limits on gas sectors per day and on fuel groups per planning horizon
  \[ \sum_{u \in U, i \in I} \left( A_{ui} g_{ui}^2 + B_{ui} g_{ui} + C_{ui} b_{ui} \right) \leq b_{ui}, \forall j \in J, d \in D. \]

– Ramp-up and ramp-down capabilities of TCUs and (H) CCPs
  \[ b_{ui} - g_{ui} + C_{ui} b_{ui} \leq b_{ui}, \forall o \in O. \]

– Minimum up and down time for TCUs and (H) CCPs
  \[ \sum_{i \in I} \beta_{ui} - \tau_{ui} \geq \tau_{ui}, \forall u \in U, i \in I. \]

– Generation output
  \[ \beta_{ui} \leq g_{ui} - g_{ui}, \forall o \in O. \]


2.2. LR–DP component-based model for (H) CCPs

The component-based model is able to individually represent the three major components in an (H) CCP, namely combustion turbine generators, heat recovery steam generators and the steam turbine generator. This allows for the inclusion of technical constraints ignored by the aggregated model such as start-up sequences, transitions between configurations and states, and the minimum/maximum time that the (H) CCP must remain in any given configuration/state. The aforementioned is achieved in [25] by using forward DP. The Lagrange multipliers needed can be obtained by means of any of the widely accepted approaches based on either cutting plane methods or sub-gradient methods and the use of various heuristics. The objective function to be minimized for each sub-problem of the DP process includes variable generation costs, variable start-up costs, transition costs between configurations and states, and all the coupling constraints. The objective function is subject to all the typical local constraints.

3. A Heuristic algorithm to solve the UC problem

When solving the UC problem, power system operators need to find an acceptable compromise between computation time and complexity of the UC model. An overly simplified UC model can produce very fast solutions but these are rendered useless when applied to real-life large-scale power systems. On the other hand, an overly complex UC model can produce excellent solutions for real-life large-scale power systems but with extremely high computation times, which is unacceptable.

Since (H) CCPs are modeled in [24] using the aggregated model, their configurations and states are ignored. Because of this, violations on transitions between configurations and states may occur. The component based model in [25] for (H) CCPs overcomes this shortcoming of the aggregated model. The main challenge faced with the LR–DP approach is that, regardless of the method used to update the Lagrange multipliers, it tends to obtain solutions that are feasible for the dual problem but infeasible for the primal problem. This problem exacerbates as the number and type of relaxed constraints increases. The advantage of using the LR–DP approach is that, the implementation of component-based models for (H) CCPs using DP is straightforward. On the other hand, representing (H) CCPs with the component-based model using an MI (L) P formulation would increase the number of integer/linear variables which, in turn, would increase the computation time. In fact, [10] points out that computation time and memory requirements are obstacles for solving real-life large-scale power systems using an MILP formulation.

Based on the above, the authors present a novel Heuristic algorithm to solve the UC problem for large-scale power systems that combines an MIQCQP formulation for the UC problem with a component-based LR–DP formulation for (H) CCPs.

The Heuristic algorithm uses various optimization techniques in order to gather the different components needed throughout the process of finding the solution; these are MILP, QP, QCP and DP. The algorithm itself is coded using Intel (R) Fortran 11.0 and IBM CPLEX 12.2.6 [6] is used as the main optimization engine. A description on how the various optimization techniques are used throughout the solution process is given next.

- **MILP:** Used to obtain the commitment (solution to the integer variables) for TCUs and a preliminary commitment for (H) CCPs.
  - Obtains a preliminary commitment using the aggregated model for (H) CCPs while their final commitment is yet to be obtained and fixed by DP using the component-based model.
- Quadratic fuel constraints, if present, are linearized.
- Start-up costs are fixed.
- Experience has shown that the commitment does not change, or changes ever so slightly, when the same problem is solved as an MILP or as an MIQP. However, the computation time and effort is greater if the problem is solved as an MIQP. Therefore, to reduce the computation time and effort, the UC problem is represented as an MILP by dropping the quadratic term the objective function.

- **QP**: Used to obtain the economic dispatch for TCUs and (H) CCPs, and the dual variables for the coupling constraints needed for the DP programming part of the algorithm.
  - The objective is quadratic.
  - Quadratic fuel constraints, if present, are linearized.
  - When the quadratic fuel constraints are linearized, the dual variables obtained for these coupling constraints are actually the dual variables of their linearized version.

- **QCP**: Used to obtain the economic dispatch for TCUs and (H) CCPs when fuel constraints are present.
  - The maximum fuel consumption constraints are considered quadratic while the minimum fuel consumption constraints are kept linearized.

- **DP**: Used to obtain the final commitment (solution to the integer variables), variable start-up costs, and variable start-up fuel consumption for (H) CCPs.
  - For (H) CCPs, DP is used to determine their commitment and obtain the variable start-up costs and variable start-up fuel consumption.
  - Following a similar approach to [26], variable start-up costs could easily be included in the MIQCP formulation at the expense of an increase in computation time. In order to reduce the computation time, and since the use of DP is needed in order to represent (H) CCPs with the aggregated model, the variable start-up costs and variable start-up fuel consumptions for TCUs are considered in the DP part of the algorithm.

The Heuristic algorithm is depicted in Fig. 1. Notice that the boxes in Fig. 1 are numbered to aid in the explanation below. To further simplify the explanation, two cases are considered: First the case without fuel constraints followed by the case with fuel constraints.

### 3.1. Without fuel constraints

1. The UC problem is solved as an MILP in Box 1 by linearizing the objective function. In this step, the (H) CCPs are represented using the simplified aggregated model.
2. The component-based model is needed in order to fully represent all the complexities of (H) CCPs. Hence, the dual variables of the UC problem are required for the DP part of the algorithm. Using the commitment obtained in Box 1, the UC problem is now solved as a QP in Box 2 in order to obtain the dual variables.
3. DP is used in Box 3, along with the dual variables obtained in Box 2, and the final commitment for the (H) CCPs is found.

![Fig. 1. Heuristic UC algorithm for large-scale Power Systems.](image-url)
this point the final variable start-up costs and the final variable start-up fuel consumption for the (H) CCPs are obtained.

4. With the commitment for the (H) CCPs fixed, the MILP representation of the UC problem is solved again in Box 1 in order to obtain the final commitment for TCUs.

5. A QP representation of the UC problem is now solved in Box 2 to obtain the final economic dispatch for TCUs and (H) CCPs, and the final set of dual variables.

6. In Box 4, the commitment for TCUs is evaluated to obtain the final variable start-up costs and final variable start-up fuel consumption.

3.2. With fuel constraints

1. Once more the UC problem is solved as an MILP in Box 1. In addition to linearizing the objective function, the quadratic fuel constraints are also linearized.

2. Using the commitment obtained in Box 1, the UC problem is now solved as a QP problem in Box 2. In order to obtain the dual variables of the coupling constraints needed in the DP part of the algorithm, the quadratic fuel constraints are linearized.

3. In Box 3 DP is used, along with the dual variables obtained in Box 2, and the final commitment for (H) CCPs using the component-based model is found. While finding the commitment of (H) CCPs, the DP algorithm also obtains the final variable start-up costs and the final variable start-up fuel consumptions.

4. With the commitment for the (H) CCPs fixed, the UC problem is solved again as an MILP in Box 1 to obtain the final commitment for TCUs.

5. With the commitment for all the units fixed, a QP problem in Box 3 is solved to obtain the final set of dual variables. Notice that the dual variables for the fuel constraints are the ones of their linearized counterparts.

6. In Box 5, the quadratic fuel constraints for the units with limited fuel supply are reintroduced and a QCP problem is solved in order to obtain the final economic dispatch for all the units.

7. In Box 4, the commitment for TCUs is evaluated to obtain the final variable start-up costs and final variable start-up fuel consumptions.

4. Numerical examples

The purpose of this section is to validate the solution approach used in the Heuristic model by comparing it to two other approaches, namely (i) to solve the UC problem directly as an MIQ (C) P and (ii) to provide a starting point to the MIQ (C) P by means of first solving an MILP. This Section also presents some numerical results when the Heuristic model is used to solve the Mexican Power System (MPS) for 1-week with scenarios with a 1-hour time window. The algorithms are coded using Intel (R) Fortran 11.0 and IBM CPLEX 12.2 is used as the main optimization engine.

The computer equipment used to solve the test systems is an HP Integrity Superdome Server with four 1.6 GHz/24 MB iL3 Intel Itanium Dual Core processors and 33 GB of RAM.

The base test system for the 1-week scenario, using representative data of the MPS, is formed by 215 TCUs, 9 CCPs, and 1 HCCP. CCPs range from two, three, and four Combustion Turbine (CT) generators and one Steam Turbine (ST) generator. The HCCP has three CT generators and one ST generator. All of them have a Heat Recovery Steam Generator (HRSG) per CT generator. There are 36 spinning reserve groups, 80 tie-lines, and 8 tie-line groups. The sensitivity flow constants for the transmission network constraints are obtained considering an electric network of 1563 nodes and...
1479 transmission lines. Only limited power flow transmission lines are taken into consideration for the calculations. The total thermal installed capacity, including must-run units, of the MPS is 50,760 MW. Thermal units can be basically divided into two different sets depending on their range of power change: 33–100% and 50–100%. The power demand supplied with thermal dispatch-able units for a typical 1-week planning scenario ranges from 22,400.00 to 30,400.00 MW.

In the next Subsection, the approach used by the Heuristic model to solve the UC problem is compared to two other approaches in order to validate the quality of its solution. Then, in the following Subsection, numerical results of the Heuristic model are presented in order to prove the superiority of its performance.

4.1. Two other ways to solve the UC problem

All the approaches described in this section use the aggregated model for (H) CCPs.

There is no doubt that the best way to solve the UC problem, in terms of global optimality and solution quality, is to directly use an MIQ (C) P approach. This is shown in Fig. 2 and it is referred to as Approach 1. What detracts from Approach 1 is the extremely high computation time needed to obtain a solution.

Another way to solve the UC problem is to first linearize the objective function and solve the problem as an MILP. If quadratic fuel constraints are present, they are also linearized. Afterwards, the original UC problem is solved as an MIQ (C) P using the solution obtained by the MILP only as a starting point. This is shown in Fig. 3 and it is referred to as Approach 2. Even though small gap values are obtained considerably faster than with Approach 1, the total computation time required to solve the UC problem is the same as with Approach 1.

Approach 3, the one used by the Heuristic algorithm, consists of two main steps: (i) Linearize the objective function and solve the UC problem as an MILP and (ii) keep this commitment and find an economic dispatch by either using QP or QCP depending on whether the problem has quadratic fuel constraints or not. The main strength of Approach 3 is that it obtains good quality solutions to the UC problem a lot faster than the other two approaches. Approach 3 is depicted in Fig. 4.

Table 1 shows the results obtained by the three different approaches discussed above when used to solve the same 1-day scenario. This 1-day scenario has the same characteristics as the 1-week scenario described above and all the constraints described in Section 2 are included except for fuel constraints. Three units have the same ramp-up and ramp-down constraints of 1 MW/min. The power output of two of them range from 140–300 MW; the other has a power output range of 300–525 MW.

Table 1 shows that the best solution is obtained by Approaches 1 and 2. Notice though, that Approach 3 obtains a solution that is only 0.15% apart from the one obtained by Approaches 1 and 2. Yet, it does so in a minimal amount of time, just 28 s, compared to the 8 h taken by the other two approaches. After hundreds of tests, the results are always consistent: Approach 3 is capable of finding solutions that are 0.1–0.3% apart from the ones obtained by Approaches 1 and 2. For 1-day scenarios, Approach 3 is able to find solutions in just a
few seconds; for 1-week scenarios, it finds solutions in just a few minutes. This clearly confirms that Approach 3 is a sound and reliable way of solving real-life large-scale power systems.

Up until now the (H) CCPs have been modeled with the aggregated model. Since transitions between configurations and states are being ignored, violations on these constraints will occur. To consider in detail (H) CCPs, the component-based model has to be used. The Heuristic model proposed in this Paper is capable of combining the component-based modeling for (H) CCPs with an MIQ (C) P formulation.

4.2. The Heuristic model

The Heuristic algorithm introduced in this Paper is part of a more complete security-constrained hydro-thermal coordination software that is currently being tested by the Mexican Power System Operator CENACE-CFE (Centro Nacional de Control de Energía–Comisión Federal de Electricidad, acronym in Spanish).

![Fig. 4. MILP used to find the commitment; Approach 3.](image)

The thermal part of this software is capable of solving the UC problem for up to 1-week scenarios with a 1-hour time window.

To further illustrate the benefits of considering (H) CCPs using the component based model, the Heuristic algorithm is compared with Approach 3. Consider the 1-week scenario described above. All the constraints described in Section 2 are included except for fuel constraints. Ramp rate constraints are as in the 1 day scenario discussed above.

Table 2 shows the comparison in costs between the Heuristic algorithm, where (H) CCPs are represented with the component-based model, and Approach 3, where (H) CCPs are represented with the aggregated model.

From Table 2 it can be seen that Approach 3 finds a cheaper solution than the Heuristic algorithm. This is because the solution found by Approach 3 has violations on the transitions between configurations and states of two CCPs. Tables 3 and 4 show the difference in commitment for one of these two CCPs.

Table 3 shows that in hour 159 the Cold Start-Up sequence (CSUS) is violated; the CCP goes from a Cold Stop (CS) to the 1 CT + ST configuration. For this particular CCP, the minimum time that it must remain in the 1 CT + ST configuration is 4 h. Therefore, there is also a violation on the minimum time for the 1 CT + ST configuration in hour 160.

From Table 4 it can be seen that since the CCP has been in a Warm Stop (WS) from hour 146 to hour 148, before the 2 CT + ST configuration is allowed, a Warm Start-Up sequence (WSUS) must be completed from hours 150 to 152. As expected, the adding of these constraints to the model results in an increase in the operation costs.

With the purpose of showing the performance of the Heuristic algorithm when fuel constraints are included, Table 5 shows the results when fuel constraints are, and are not present. The test results for Approach 3.
system is the 1-week scenario described above. For both instances, with and without fuel constraints, ramp rate constraints are included. For the case, where fuel constraints are included, one fuel group and one gas sector have been limited.

From Table 5 one can see that since the fuel constraints are binding, the costs obtained are higher than the case with no fuel constraints. Notice that in both instances the Heuristic algorithm was able to obtain a solution in under twenty minutes. Notice also that the inclusion of quadratic fuel constraints did not have an important impact on the computation time. The performance of the Heuristic algorithm is outstanding since, for this particular case, the optimization model solved contains over 240,000 variables (integer and continuous), and over 244,000 constraints (quadratic and linear).

5. Conclusion

The authors present a novel Heuristic algorithm to solve the UC problem for real-life large-scale power systems. This algorithm is capable of solving a complex UC model with very attractive computation times. It includes quadratic fuel constraints that are often ignored in other UC models. The Heuristic model successfully combines MILP, QP, QCP, and DP in order to benefit from global convergence capabilities and a detailed representation of (H) CCPs. The numerical results prove that the solutions found by the Heuristic algorithm are in fact good quality solutions since they are only 0.1–0.3% apart from the ones obtained by the more rigorous approaches. The Heuristic algorithm introduced here is a timely answer to the needs of power system operators in terms of solution quality and computation time.

Appendix A. Notation

The following is the notation used in the models presented in this paper.

Indices and Sets

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>i, j</td>
<td>Indices for intervals in the planning horizon</td>
</tr>
<tr>
<td>i, u</td>
<td>Indices for intervals in the planning horizon</td>
</tr>
<tr>
<td>CH</td>
<td>Set of Hybrid Combined Cycle Plants</td>
</tr>
<tr>
<td>CT</td>
<td>Set of interruptible loads in the system</td>
</tr>
<tr>
<td>D</td>
<td>Set of days in the planning horizon</td>
</tr>
<tr>
<td>GH</td>
<td>Set of gas units pertaining to Hybrid Combined Cycle Plants</td>
</tr>
<tr>
<td>T</td>
<td>Set of intervals in the planning horizon</td>
</tr>
<tr>
<td>J</td>
<td>Set of gas sectors</td>
</tr>
<tr>
<td>M</td>
<td>Set of regional tie-lines</td>
</tr>
<tr>
<td>N</td>
<td>Set of regional tie-line groups</td>
</tr>
<tr>
<td>O</td>
<td>Set of fuel groups</td>
</tr>
<tr>
<td>PE</td>
<td>Set of Independent Power Producers</td>
</tr>
<tr>
<td>R</td>
<td>Set of spinning reserve groups</td>
</tr>
<tr>
<td>U</td>
<td>Set of all units in the system</td>
</tr>
<tr>
<td>V</td>
<td>Set of steam units pertaining to Combined Cycle Plants</td>
</tr>
</tbody>
</table>
| Z | equals $T \cup S \cup G H$
| Z' | equals $S \cup V H$

Constants

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_{u,i}$</td>
<td>Quadratic coefficient of the cost function for $u$ in $i$; in $$/MW$^2$ h</td>
</tr>
<tr>
<td>$A_{u,i}$</td>
<td>Quadratic coefficient of the fuel consumption function for $u$ in $i$; in GCal/MW$^2$ h</td>
</tr>
<tr>
<td>$AC_{u,i}$</td>
<td>Hot start cost for $u$ in $i$; in $$/h</td>
</tr>
<tr>
<td>$AF_{u,i}$</td>
<td>Cold start cost for $u$ in $i$; in $$/h</td>
</tr>
<tr>
<td>$b_{u,i}$</td>
<td>Linear coefficient of the cost function for $u$ in $i$; in $$/ MW h</td>
</tr>
<tr>
<td>$b_{u,i}$</td>
<td>Linear coefficient of the fuel consumption function for $u$ in $i$; in GCal/MW h</td>
</tr>
<tr>
<td>$b_{j,d}$</td>
<td>Maximum and minimum value of gas consumption for $j$ in $d$, respectively; in GCal</td>
</tr>
<tr>
<td>$b_{o,d}$</td>
<td>Maximum and minimum value of fuel consumption for $o$, respectively; in GCal</td>
</tr>
<tr>
<td>$c_{u,i}$</td>
<td>Constant term of the cost function for $u$ in $i$; in $$/h</td>
</tr>
<tr>
<td>$C_{u,i}$</td>
<td>Constant term of the fuel consumption function for $u$ in $i$; in GCal/h</td>
</tr>
<tr>
<td>$ce_{h,i}$</td>
<td>Sum of products between the sensitivity flow and constant power injections in $m$ for $i$; in MW</td>
</tr>
<tr>
<td>$CC_i$</td>
<td>Fixed cost for load shedding in $i$; in $$/MW h</td>
</tr>
</tbody>
</table>

Table 5

<table>
<thead>
<tr>
<th>Fuel const.</th>
<th>Total cost</th>
<th>Production cost</th>
<th>Start-up cost</th>
<th>CPU time</th>
</tr>
</thead>
<tbody>
<tr>
<td>3,449,815,793</td>
<td>3,446,086,751</td>
<td>3,729,042</td>
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<td>3,595,125,748</td>
<td>3,591,808,382</td>
<td>3,317,366</td>
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</table>

Indices for units in the system:

- $ch$ Index for Hybrid Combined Cycle Plants
- $d$ Index for days in the planning horizon
- $i$, $j$ Indices for intervals in the planning horizon
- $l$ Index for interruptible loads
- $m$ Index for fuel groups
- $n$ Index for regional tie-line groups
- $o$ Index for spinning reserve groups
- $u$, $vh$ Indices for units in the system

Table 4

<table>
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<tr>
<th>Hour</th>
<th>CT 1</th>
<th>CT 2</th>
<th>Res</th>
<th>CCP</th>
<th>Conf.</th>
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</thead>
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<tr>
<td>145</td>
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<td>37.0</td>
<td>80.0</td>
<td>154.0</td>
<td>2 CT + ST</td>
</tr>
<tr>
<td>146</td>
<td>0.0</td>
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<td>0.0</td>
<td>0.0</td>
<td>WS</td>
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<tr>
<td>147</td>
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<td>0.0</td>
<td>WS</td>
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<tr>
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<td>0.0</td>
<td>0.0</td>
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<tr>
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<td>0.0</td>
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<tr>
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<td>154.0</td>
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<td>0.0</td>
<td>CS</td>
</tr>
</tbody>
</table>

A In $.$
B In min.
\[ c_{g_{u,i}} \] Sum of products between the sensitivity flow and constant power injections in \( i \) for \( i \) in MW

\[ C_{I,i} \] Variable cost for interruptible loads for \( I \) in \$/MW h

\[ CP_{u,i} \] Variable cost for Independent Power Producer \( u \) in \$/MW h

\[ d_i \] Demand in \( i \) in MW

\[ f_{n_{m,i}} \] Maximum value for the power counterflow on \( m \) in \( i \) in MW

\[ f_{n_{n,i}} \] Maximum value for power counterflow on \( n \) in \( i \) in MW

\[ f_{P_{m,i}} \] Maximum value for the power flow on \( m \) in \( i \) in MW

\[ f_{P_{n,i}} \] Maximum value for the power flow on \( n \) in \( i \) in MW

\[ g_{f_{\text{Fixed wrap}} u,i} \] Fixed generation of \( u \) in \( i \) in MW

\[ g_{u,0} \] Initial generation of \( u \) in \( i \) in MW

\[ g_{u,i} \] Maximum and minimum generation value of \( u \) in \( i \), respectively; in MW

\[ g_{u,i}^+ \] equals \( g_{u,i}(1 + a_{u,i}) \); in MW

\[ g_{u,i}^- \] equals \( g_{u,i}(1 - a_{u,i}) \); in MW

\[ I_{r,i} \] Spinning reserve of \( r \) in \( i \) in MW

\[ l_{u,i} \] Maximum and minimum value of curtailed load for \( l \) in \( i \) in MW

\[ p_m \] Fixed cost of deviation from the limits of \( m \) in \$/MW h

\[ p_n \] Fixed cost of deviation from the limits of \( n \) in \$/MW h

\[ R_{R_{u}} \] Ramp-down rate of \( u \) in MW/h

\[ R_{S_{u}} \] Ramp-up rate of \( u \) in MW/h

\[ s_{F_{\text{Standalone}} u,i} \] Sensitivity flow in \( m \) w.r.t changes in power injections of \( u \) in \( i \); dimensionless

\[ s_{G_{\text{Standalone}} u,i} \] Sensitivity flow in \( n \) w.r.t changes in power injections of \( u \) in \( i \); dimensionless

\[ t_{u,i} \] Number of intervals that \( u \) has been off up to \( i \) in h

\[ t_{u} \] Minimum up time for \( u \) in h

\[ t_{u,i} \] Minimum down time for \( u \) in h

\[ a_{u,i} \] Contribution factor of \( u \) in \( i \); dimensionless

Variables

\[ f_{m,i} \] Power flow surplus on \( m \) in \( i \) in MW

\[ f_{n,i} \] Power counterflow surplus on \( m \) in \( i \) in MW

\[ f_{m,i}^- \] Power flow surplus on \( n \) in \( i \) in MW

\[ f_{n,i}^- \] Power counterflow surplus on \( n \) in \( i \) in MW

\[ g_{u,i} \] Generation of \( u \) in \( i \) in MW

\[ g_{v,h} \] Generation of \( v \) in \( h \) in MW

\[ h_{i,j} \] Load shed in the system in \( i \) in MW

\[ a_{u,i} \] Start-up of \( u \) in \( i \); binary variable

\[ \beta_{i,j} \] Commitment status of \( v \) in \( i \); binary variable

\[ \gamma_{u,i} \] Commitment status of \( v \) in \( i \); binary variable

\[ \zeta_{u,i} \] Warm stop of \( u \) in \( i \); binary variable

\[ \tau_{u,i} \] Hot stop of \( u \) in \( i \); binary variable

References


